3.40.1. Derived Rule Problems

(*Problems 1 and 2 show that the rule* Repetition *can be derived from other rules in our system – hence that* Repetition *can be treated as a* ***derived*** *rule*.)

1. Provide a deduction for the following argument, using only ~–, and ~+.

1. P

∴ P

2. Provide a deduction for the argument in Problem, 1 using only **ID**, ~+, ∨+, and ∨–.

(*Problems 3 through 5 show that the rule ~– can be derived from other rules in our system – hence that ~– can be treated as a* ***derived*** *rule*.)

3. Provide a deduction for the following argument, using only **ID** and **R**.

1. ~~P

∴ P

4. Provide a deduction for the argument in Problem 3, using only **ID** and **two instances** of ~+.

5. Provide a deduction for the argument in Problem 3, using only **ID** and **one instance** of ~+. (*Hint: use an ID within an ID.*)

(Problems 6 and 7 show that the rule ~+ can be derived from other rules in our system – hence that ~+ can be treated as a ***derived*** *rule*.)

6. Provide a deduction for the following argument, using only **ID**, ~–, and **R**.

1. P

∴ ~~P

7. Provide a deduction for the following argument, using only **ID** and **R**. (*Hint: use an ID within an ID.*)

1. P

∴ ~~P

(*Problems 8 and 9 show that with Inward DeMorgan's Law added to our deductive system, the rules of ∧+ and ∨+ can treated as* ***derived rules***.)

Inward DM

~(● ∨ ▲)	~(● ∧ ▲)
(~● ∧ ~▲)	(~● ∨ ~▲)

8. Provide a deduction for the following argument, using only **ID**, ~+, ∨–, **R**, and **inward DM**.[1]

1. P
2. Q

∴ (P ∧ Q)

9. Provide a deduction for the following argument, using only **ID**, ∧–, **R**, and **inward DM**.

1. P

∴ (P ∨ Q)

[1] As Problem 7 shows, we could eliminate use of ~+ in this deduction – at the expense of having an ID within an ID within an ID.

(*Problems 10 and 11 show that with Outward DeMorgan’s Law added to our deductive system, the rules of* $\wedge$– *and* $\vee$– *can treated as* ***derived rules***.)

Outward DM

$\dfrac{(\sim\bullet \wedge \sim\blacktriangle)}{\sim(\bullet \vee \blacktriangle)}$	$\dfrac{(\sim\bullet \vee \sim\blacktriangle)}{\sim(\bullet \wedge \blacktriangle)}$

10. Provide a deduction for the following argument, using only **ID**, $\vee$**+**, **R**, and **outward DM**.

1. $(P \wedge Q)$

$\therefore$ P

11. Provide a deduction for the following argument, using only **ID**, $\wedge$**+**, **R**, and **outward DM**.

1. $(P \vee Q)$
2. $\sim P$

$\therefore$ Q

12. Provide a deduction for the following argument using **outward DM** (along with one of the original seven rules of inference), but **without using ID**.

1. $\sim P$

$\therefore \sim(P \wedge Q)$